

One-way quantum computing in optical lattices with many atom addressing

Timothy P. Friesen and David L. Feder

*Department of Physics and Astronomy and Institute for Quantum Information Science,
University of Calgary, Calgary, Alberta, Canada T2N 1N4*

(Dated: February 1, 2008)

One of the fundamental conditions for one-way quantum computation (1WQC) is the ability to make sequential measurements on isolated qubits that comprise the highly entangled resource for 1WQC, the cluster state. This has been a significant impediment in the implementation of 1WQC with ultracold atoms confined in optical lattices, because the width of the measuring lasers is generally much greater than the atomic (qubit) spacing. We demonstrate that deterministic 1WQC is nevertheless possible, with a polynomial increase in the number of operations, as long as the center of the beams can be positioned with high accuracy. Extending the number of cluster atoms, the scheme is also able to compensate for accidental measurements of an arbitrary number of nearby qubits.

PACS numbers: 03.67.Lx, 03.75.Ss, 05.30.Jp

I. INTRODUCTION

In one-way quantum computing (1WQC), first proposed by Raussendorf and Briegel [1], the computation proceeds entirely by performing a sequence of single qubit measurements on a special many qubit entangled state, known as a cluster state. This approach is wholly equivalent to the quantum circuit model, but provides two distinct practical advantages. First, all of the required entanglement is generated at the outset in only two steps, involving all of the particles simultaneously. Second, no entangling gates are required between arbitrary (and potentially distant) qubits during the computation. The 1WQC approach therefore lends itself to any physical architecture where one can induce genuine multipartite entanglement and perform single-qubit measurements. To date, 1WQC has been demonstrated in linear optical systems by the implementation of Grover's search algorithm for up to four qubits [2].

Ultracold atoms confined in optical lattices arguably constitute the most promising test beds for quantum simulation and computation, and are natural candidates for the implementation of 1WQC. This optimism is due to the exceptional control of long-lived atomic internal states, the lattice parameters, and the interactions between particles. Under the right conditions, exactly one bosonic atom will occupy each site of the optical lattice at low temperatures as the gas undergoes a quantum transition into a Mott insulating phase [3]. By adjusting the orientation and strength of the applied lasers, the Mott phase can be realized for atoms in effective one-dimensional (1D), 2D, or 3D lattices [4]. Because two internal states can be chosen as computational registers, each atom corresponds to a qubit. In the Mott phase, these (neutral) qubits are effectively stationary and non-interacting. The massively entangled cluster state that is the central resource for 1WQC can then be generated with relative ease [5, 6, 7]: starting with a single large quasi-2D array of atoms [8, 9], entanglement between neighboring atoms can be done in parallel with state-

dependent collisions [10, 11] or with tunable spin-spin interactions [12, 13]. Allowing these operations to occur for the right amount of time on qubits initialized in the zero eigenstate of the Pauli X operator, one can implement (up to local unitaries) the maximally entangling controlled-phase (CZ) gate that generates the cluster state.

While the relative ease with which one may generate large cluster states containing thousands to millions of physical qubits appears to strongly favor ultracold atoms in optical lattices, these systems tend to suffer from one important shortcoming: the practical difficulty of measuring single qubits. Most optical lattice experiments to date utilize ^{87}Rb atoms [3, 11], with a resonant absorption at a wavelength of 780 nm. The wavelengths of the lasers forming the optical lattice are generally chosen to be sufficiently close to this resonance (usually $\lambda \approx 800$ nm) in order to yield strong confinement while minimizing spontaneous emission which causes heating and loss of atoms. Assuming perfectly counter-propagating lasers, the spacing between sites of the optical lattice is therefore $a \approx 0.4 \mu\text{m}$. Yet the additional lasers that would be used to apply rotations or measurements on selected qubits can generally be focused to widths on the order of $3 - 4 \mu\text{m}$, which is a few times the fundamental resolution limit.

Much effort in recent years has been devoted to improving the addressability of single qubits in optical lattices. An experimental scheme has been realized wherein atoms occupying every site of an optical lattice are transferred to every third site [14]; in principle this process can be repeated until the desired separation is reached. A CO_2 laser generates an optical lattice with the long spacing $a = 5.3 \mu\text{m}$ [15]; while a Mott transition in this lattice is difficult to achieve, unit-filling in a finite region may be reached by physically moving atoms through state-dependent laser manipulation [16]. This kind of state-dependent transport can also be used to move a small number of 'marker' atoms through the lattice, generating entanglement with the cluster qubits [17, 18, 19];

measurements of the (well-separated) marker atoms can then be unambiguously made. A recent theoretical proposal makes use of the energy shifts induced by additional lasers to ensure that only one of the atoms is resonant with the measuring laser [20]. Another uses interference patterns generated by multiple additional lasers oriented at different angles to localize atoms [21]; unfortunately, it is generally not feasible experimentally to generate beams from very many directions or to have the attendant optical access. A very recent proposal is to pump atoms in the vicinity of the target into internal states that are decoupled from the subsequent manipulations [22].

In the present work we show that high-fidelity 1WQC does not in fact require single-qubit measurements, and in principle is experimentally feasible with lasers that simultaneously impinge on a large number of atoms from the same direction. The spatial variation of the Gaussian measuring beam is the crucial ingredient making this possible. In general, projective measurements on multiple qubits generally yield mixed states because the lack of spatial resolution prevents knowing which qubits were projected into the computational basis states $|0\rangle$ and $|1\rangle$. In order to reduce the uncertainty, one could simply re-measure different subsets of the qubits. In this scenario one would expect to require $N - 1$ additional measurements for each initial projection of N qubits to reduce the uncertainty to zero and yield a pure state. An even simpler approach would be to arrange that the states of all qubits to be irradiated (except the one of interest) are in a simple fiducial state, such as one of the computational basis vectors, so that the ambiguity in the result of the multi-qubit measurement is always zero. In this second scenario, one would expect $N - 1$ additional unitary operations in advance of each measurement. As discussed in detail below, it is always possible in principle to implement the second approach, ensuring high-fidelity 1WQC even for large numbers of simultaneously measured qubits.

The manuscript is organized as follows. The basic ideas of 1WQC and the fundamentals of implementing rotations and measurements for atoms in optical lattices are reviewed in Section II. The details of how to initialize atoms in the states required to form one-dimensional cluster states with many atom rotations are described in Section III, and the protocol for performing (non-universal) 1WQC with multiple measurements of atoms on a line is discussed in Section IV. The full two-dimensional protocol is described in Section V. The results are summarized in Section VI, with comments on how the work could be extended.

II. 1WQC AND SINGLE ATOM OPERATIONS

A. One-way Quantum Computing

Starting with a periodic two dimensional array of N qubits, a cluster state can be prepared by first initializing

all N qubits to the $+1$ eigenstate of the Pauli X operator $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, followed by the application of controlled-phase $CZ = \text{diag}(1, 1, 1, -1)$ gates between all nearest neighbors. Computational measurements, made in the eigenbasis of the Pauli Z operator, are then performed to remove qubits from the initial cluster state and to enable the construction of a computation-specific cluster state (hereafter denoted a computational cluster state) with the desired connections. In the simplest (though not most computationally efficient) approach, the resulting graph state is a spatial representation of a quantum circuit: horizontal chains of entangled physical qubits encode a single computational qubit, and vertical links represent two-qubit entangling operations.

Actual computation proceeds via quantum teleportation by sequentially measuring the state of each qubit in a basis defined by the states $|\pm_\xi\rangle = (|0\rangle \pm e^{-i\xi}|1\rangle)/\sqrt{2}$. Alternatively, the state of each qubit is rotated by an angle ξ around the Z axis, followed by an X -basis measurement. The X measurement can be effected by first applying a Hadamard H operation which rotates from the X basis to the Z basis, followed by a computational basis measurement. In a 1D cluster, measuring the first qubit initially in the state $|\psi\rangle$ teleports the modified state $X^m H R_z(\xi)|\psi\rangle$ to its nearest neighbor, where we define $R_\sigma(\xi) = e^{-i\frac{\xi}{2}\Sigma}$ with $\sigma = x, y, z$ and $\Sigma = X, Y, Z$. The measurement outcomes $m = 0$ and 1 correspond to projection into the computational states $|0\rangle$ and $|1\rangle$, respectively. Universal single-qubit operations can be decomposed into three successive rotations around orthogonal axes (the Euler angles), accomplished in the one-way model by three successive measurements:

$$\begin{aligned} |\psi'\rangle &= X^{m_3} H R_z(\theta_3) X^{m_2} H R_z(\theta_2) X^{m_1} H R_z(\theta_1) |\psi\rangle \\ &= X^{m_3} Z^{m_2} X^{m_1} H R_z(\theta'_3) R_x(\theta'_2) R_z(\theta_1) |\psi\rangle, \end{aligned} \quad (1)$$

where $\theta'_2 = (-1)^{m_1}\theta_2$ and $\theta'_3 = (-1)^{m_2}\theta_3$. Note that $R_x(\theta) \equiv \exp(-i\theta X)$. The Hadamard and the byproduct unitaries (the X and Z in Eq. (1) above) arising from measurement outcomes $m_i = 1$ can wait to be applied only at the end of the gate teleportation. Crucially the angles θ'_i for subsequent measurements must be adapted based on the result of previous measurements for the computation to be deterministic, a property known as ‘feed-forward.’ Together with the vertical links in the computation-specific cluster state, representing CZ gates between computational qubits, measurements in 1WQC generate a set of gates that are universal for quantum computation.

B. Single Atom Rotations

Rotations on atoms are effected either by a radio-frequency pulse resonant on the $|0\rangle \leftrightarrow |1\rangle$ transition or by a Raman transition between these levels using two lasers. In either case, the electromagnetic field associated with the radiation gives rise to the Hamiltonian in the dipole

and rotating wave approximations [23]:

$$H_I = \frac{\hbar\Omega}{2} (|1\rangle\langle 0|e^{i\varphi} + |0\rangle\langle 1|e^{-i\varphi}), \quad (2)$$

where Ω and φ are the Rabi frequency and laser phase, respectively. The time evolution operator $U = e^{-iHt/\hbar}$ is then

$$U(\Omega t, \varphi) = \begin{pmatrix} \cos(\frac{\Omega t}{2}) & -ie^{-i\varphi}\sin(\frac{\Omega t}{2}) \\ -ie^{i\varphi}\sin(\frac{\Omega t}{2}) & \cos(\frac{\Omega t}{2}) \end{pmatrix}, \quad (3)$$

where $U(\Omega t, 0) = R_x(\Omega t)$ and $U(\Omega t, \pi/2) = R_y(\Omega t)$.

Using the identity $H = -iR_x(\pi)R_y(\pi/2)$ we can write $HR_z(\theta) = R_x(\theta)H = R_x(\theta + \pi)R_y(\pi/2)$ neglecting the overall phase. Because an $R_y(\pi/2)$ is always present independent of the angle θ , one can simultaneously apply $R_y(\pi/2)$ with a wide laser beam to all the atoms in the computational cluster state before making any measurements. In this way only R_x rotations need to be applied to the qubits before measuring in the computational basis: the result of the teleportation is the same as if the rotation had been applied after measurement of the previous qubit.

The focused rotation laser is unfortunately not as narrow as the lattice spacing, and will therefore irradiate atoms near the qubit of interest. The beam intensity has a Gaussian distribution $I = I_0 \exp(-2x^2/r^2)$ where x is the distance from the center and r is the beam radius at which $I = I_0 e^{-2}$. The Rabi frequency Ω is proportional to the electric field and therefore the square root of the intensity $\Omega(x) = \Omega_0 \exp(-x^2/r^2)$. All neighboring atoms are influenced by the field for the same pulse length t , so atoms further from the beam center will undergo a rotation by an exponentially decreasing angle.

C. Single Atom Measurements

Measuring the internal state of an atom is done by fluorescence, a technique used for quantum jump detection [24, 25, 26]. Atomic measurements of this type have been experimentally demonstrated with ions (where the technique is also known as electron shelving) and work with high probability [27]. In a fluorescence measurement a pulse is applied between the computational state $|0\rangle$ and an unstable auxiliary state $|2\rangle$ which rapidly decays spontaneously back to $|0\rangle$ (the decay pathway $|2\rangle \rightarrow |1\rangle$ is forbidden). The atom will repeatedly transition between $|0\rangle$ and $|2\rangle$ producing many fluorescent photons as long as the measurement pulse is active, while an atom in $|1\rangle$ will not be affected by the measurement beam. A strong measurement corresponds to the projection of the atom's internal state wavefunction into one of the computational basis states with high certainty.

While the measurement pulse is active, the Hamiltonian for the system in the dipole and rotating wave approximations is given by:

$$H = \frac{\hbar\Omega}{2} (|0\rangle\langle 2| + |2\rangle\langle 0|). \quad (4)$$

where Ω is the $|0\rangle \leftrightarrow |2\rangle$ Rabi frequency. The time evolution of the atomic density matrix is described by the master equation [23]

$$\dot{\rho} = -\frac{i}{2}[H, \rho] + \frac{\gamma}{2}(2|0\rangle\langle 2|\rho|2\rangle\langle 0| - |2\rangle\langle 2|\rho - \rho|2\rangle\langle 2|) \quad (5)$$

where $\gamma \gg \Omega$ is the rapid decay rate from excited level $|2\rangle$. For times long compared to $1/\gamma$ one can effectively eliminate the population in $|2\rangle$. Neglecting terms proportional to $(\Omega/\gamma)^2$ one obtains

$$\dot{\rho}_{00} = 0; \quad (6)$$

$$\dot{\rho}_{11} = 0; \quad (7)$$

$$\dot{\rho}_{01} = -\frac{\Omega^2}{2\gamma}\rho_{01}. \quad (8)$$

These equations describe a two-level system with a decay of the ρ_{01} coherence a rate $\frac{\Omega^2}{2\gamma}t$. A successful measurement requires the vanishing of the off-diagonal elements (coherences) of the density matrix, $\frac{\Omega^2}{2\gamma}t \gg 1$, which together with $\Omega/\gamma \ll 1$ requires $t \gg \Omega^{-1}$. The probability of a measurement occurring is therefore given by

$$p = 1 - e^{-\frac{\Omega^2}{2\gamma}t} \quad (9)$$

which approaches unity exponentially for high intensities, strong coupling, rapid decay times, and long measurements.

As was the case for rotations, the spatial profile of the measuring laser ensures that atoms near the qubit of interest will have a reasonably high (though exponentially decreasing) probability of being measured. The consequences would appear to be disastrous for 1WQC, because unwanted projections of qubits in the computational cluster will completely destroy the state of the logical qubit. Furthermore, an inadvertent measurement will be nearly impossible to detect. The fluorescence is proportional to the local beam intensity, so that atoms projected by the low-intensity wings of the Gaussian will emit comparatively few photons. The solution to this apparent impasse will be discussed in Section IV.

III. CLUSTER STATE CREATION

The first hurdle facing practical 1WQC in an optical lattice system is the formation of the computational cluster state. The usual approach taken is to first create a standard cluster state and then to selectively remove unwanted physical qubits through Z measurements. This method is not feasible for wide measuring beams, however. There will always be some probability of inadvertently (and unknowingly) measuring the states of nearby qubits, thereby removing atoms that should form part of the cluster. A more suitable construction is to first rotate all computational cluster qubits to $|+\rangle$ and all unwanted qubits to $|0\rangle$, and only then perform the controlled phase

gates between all neighboring qubits. Qubits in $|0\rangle$ will be left untouched by the CZ operations and therefore will not be connected to neighboring qubits.

It is possible to specify the states of individual qubits in an optical lattice even for wide rotation lasers that irradiate multiple atoms. One requires only that the *center* of the beam can be positioned with high accuracy. Consider an array of N qubits on which one performs N rotation pulses, each centered on a different qubit, with angles θ_n about the same axis, where $n = 1, 2, \dots, N$. The beams have a Gaussian intensity profile and therefore will rotate each qubit m by an angle $\phi_m = \theta_n e^{-x_{mn}^2/r^2}$, where $x_{mn} = |x_n - x_m|$ is the distance from the central qubit n . This will lead to a linear system of equations with N unknowns (the θ_n) that yield the desired phases (the ϕ_n) on each qubit. In general, the linear system takes the form

$$A\vec{\theta} = \vec{\phi} \quad (10)$$

where A is of the form

$$A = \begin{pmatrix} 1 & a_{12} & a_{13} & a_{14} & \dots & \cdot \\ a_{21} & 1 & a_{23} & \dots & \cdot & \\ a_{31} & a_{32} & 1 & \dots & \cdot & \\ \cdot & & & \cdot & & \\ \cdot & & & & \cdot & \\ \cdot & & & & & \cdot \end{pmatrix}, \quad (11)$$

$$a_{nm} = e^{-x_{nm}^2/r^2} = a_{mn}, \quad (12)$$

$$\vec{\theta} = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}, \quad \vec{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}. \quad (13)$$

In order for a solution to exist one requires

$$\det(A) = \sum_k b_k e^{-k/r^2} \neq 0, \quad (14)$$

where both the k (sums of distances squared in the x and y directions of the cluster) and the b_k (combinatoric coefficients) are integers. Suppose one assumes that $\det(A) = 0$. This is equivalent to stating that e^{-1/r^2} is a root of a polynomial with integer coefficients. For rational $-1/r^2$, however, e^{-1/r^2} is transcendental which by definition cannot be the root of a polynomial with integer coefficients [28]. It follows that $\det(A)$ is non-zero and a solution exists for the linear equations (10). Therefore, with one beam centered on each of the N atoms it is possible to precisely control the rotation applied to each individual atom.

Consider a simple example in order to demonstrate the procedure. Suppose there are three qubits oriented as



FIG. 1: Building a two-qubit cluster state with three qubits. The initial state (a) has all three qubits in state $|+\rangle$; after rotations qubit 3 is in state $|0\rangle$, so that entanglement leaves the desired final state (b).

shown in Fig. 1, and one wishes to generate entanglement between the qubits 1 and 2 but leave qubit 3 disentangled, Fig 1(b). First all three qubits are initialized to $|+\rangle$ with a very wide beam (which assumes negligible spatial variation in the intensity). Three $R_y(\theta_i)$ pulses are then applied, each one centered on a different qubit, such that qubits 1 and 2 remain in $|+\rangle$ while qubit 3 returns to $|0\rangle$. The linear system is:

$$1 : \theta_1 + e^{-1/r^2} \theta_2 + e^{-1/r^2} \theta_3 = 0; \quad (15)$$

$$2 : e^{-1/r^2} \theta_1 + \theta_2 + e^{-2/r^2} \theta_3 = 0; \quad (16)$$

$$3 : e^{-1/r^2} \theta_1 + e^{-2/r^2} \theta_2 + \theta_3 = -\pi/2, \quad (17)$$

which has the solution

$$\theta_1 = \frac{\pi}{2} \frac{e^{-1/r^2}}{1 - e^{-2/r^2}}; \quad (18)$$

$$\theta_2 = 0; \quad (19)$$

$$\theta_3 = -\frac{\pi}{2} \frac{1}{1 - e^{-2/r^2}}. \quad (20)$$

In this particular case only two rotations are actually needed. Note that the solution depends on the beam radius r only in terms of an (experimentally) adjustable parameter, as long as r remains finite. For a larger number of qubits and a wide beam, each rotation will affect many qubits across the lattice, but as long as there are as many distinct operations as qubits, each applied rotation may be chosen such that the overall phase on each qubit is exactly as desired.

IV. 1D CLUSTER COMPUTATION

A. Offset Measurements

The simplest way to reduce the probability of inadvertently measuring qubits in an entangled chain is to offset the center of the measuring beam. Consider the very first qubit in a one-dimensional cluster, Fig. 2. To the left of this qubit there is empty space and to the right are connected qubits. We wish to measure the first qubit with high probability while preserving the states of its neighbors. This can be accomplished by taking advantage of the Gaussian shape of the laser pulse: shifting the beam to the left while increasing its maximum intensity

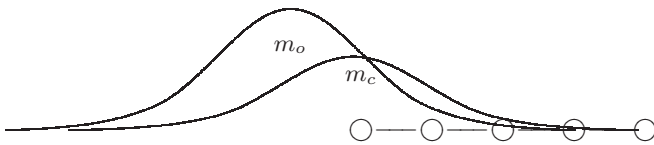


FIG. 2: The advantage of offsetting Gaussian pulses. The centered beam M_c has the same intensity on the target qubit as a higher-intensity beam M_o whose center is displaced from the target qubit by a single lattice spacing to the left, but M_o has an exponentially reduced intensity on the first neighbor to the right.

reduces its amplitude on the neighboring qubits without altering its effect on the target atom.

Suppose that one requires intensity $I = I_t$ on the first (target) qubit. Shifting the beam center n lattice spacings a to the left yields intensity on the target $I = I_n e^{-2(na)^2/r^2}$, so that the intensity maximum of the beam must increase exponentially in the offset: $I_n = I_t e^{2(na)^2/r^2}$. The intensity on the first neighbor to the right of the target I_f nevertheless decreases exponentially in the offset: $I_f = I_t e^{2(na)^2/r^2} e^{-2(n+1)^2 a^2/r^2} = I_t e^{-2(2n+1)a^2/r^2}$. If the desired probabilities of measuring the target and nearest-neighbor qubits are p_t and p_f , respectively, then from Eq. (9) one obtains the required offset

$$n = \frac{r^2}{4a^2} \ln \left[\frac{\ln(1 - p_t)}{\ln(1 - p_f)} \right] - \frac{1}{2}. \quad (21)$$

For example, if $p_t = 0.99$ and $p_f = 0.01$ were desired, then with $r = 4a$ one would require $n \approx 24$. With $r = 10a$ the offset jumps to approximately $153a$. The probability of simultaneously measuring the second nearest neighbor p_s is truly negligible:

$$p_s = 1 - \exp \left[e^{-4a^2/r^2} \frac{\ln^2(1 - p_f)}{\ln(1 - p_t)} \right]. \quad (22)$$

For the same choices of p_t and p_f above, one obtains $p_s \approx 10^{-5}$.

The discussion above has assumed that the target qubit is located at the left-most edge of the chain, with its neighbors to the right. In the most general case of an offset measurement, however, a significant number of qubits M to the left of the target will also be measured at each stage of the computation ($M \approx 2n$ where na is the offset distance discussed above). Because all of these were target qubits during previous steps in the one-way computation, their subsequent projections into computational basis states during the measurement of the target qubit will not affect the logical states of any of the cluster qubits. That said, rotation pulses will have been applied to these qubits by the wide lasers during the computation, so their measurement outcomes are not predetermined.

One must be able to clearly distinguish the fluorescence of the target atom from that of all the qubits to its left when performing a measurement with an offset beam. The simplest solution is to re-set the states of all M non-cluster qubits that will be re-measured to the non-fluorescing state $|1\rangle$. This can be accomplished with the same method as described in Sec. III to initialize all of the qubits' states. The phases φ_i accumulated on the M relevant qubits by previous rotations are known, so that one can choose $\phi_i = -\varphi_i + \pi$ as input to the linear system of equations to find the θ_i . Note that the application of the M rotations will also build up an undesired phase on the target qubit, so this qubit needs to be included in the linear system, with $\phi_{\text{target}} = 0$. Thus, $M+1$ operations are required for each target measurement. In addition, the phases of $2n$ qubits need to be calculated classically at each stage; together with the time for the classical solution to the $2n+1$ -dimensional linear system, each measurement has a classical overhead that scales like $4n^2$.

B. Measurement Protocol

As discussed in detail above, with wide beams one is liable to make inadvertent measurements of atoms close to the target qubit, though the probability of this occurring can be made arbitrarily small in principle by offsetting the center of the measuring beam from the target. If only one of the measured qubits to the right of the target is in a state outside the XY plane prior to its measurement, the logical qubit will undergo a non-unitary (and therefore uncorrectable) transformation. The most extreme case is a Z measurement, which unentangles the measured qubit from the rest of the cluster and destroys the entanglement resource needed for quantum teleportation. Even if all measured qubits' states were initially in the XY plane, the absence of feed-forward makes it unlikely that the desired unitary transformation will be implemented.

There are two cases when feed-forward is not required, however. The first is if all of the measurement outcomes are zero, in which case there are no byproduct operators to commute through the applied rotations, and therefore no compensation of the choice of measurement angle. Unfortunately, the likelihood of this occurrence decreases exponentially with the number of measured qubits, and in any case the results of inadvertent measurements are by definition unknown. The second case corresponds to teleportation of single qubit Clifford operations, which byproduct operators can commute through without changing the operation [29]. It is impossible, however, to effect universal quantum computation using Clifford operations alone.

Below we describe in detail a protocol that eliminates errors arising from inadvertent measurements of near neighbors, with linear overhead in terms of ancillary physical qubits and operations. Feed-forward is normally

accounted for by applying a corrective rotation depending on the result of the previous measurement. As discussed in Sec. II A, this corrective rotation need only be applied when that result is one. When $m_1 = 0$ the gate teleportation is successful even if an inadvertent measurement of a neighbor has occurred. When $m_1 = 1$ there will be some probability p that an error has occurred, but the negative consequences of this can be avoided by implementing feed-forward in an alternative manner. Rather than applying a corrective rotation on the next measurement, we resign ourselves to rotating in the wrong direction. Instead, the sequence of measurements is extended by a number of qubits on which the phase error can hopefully be corrected. We first describe in detail the protocol to compensate for inadvertent measurements of the qubit that is the nearest-neighbor of the target; the extension to the case where an arbitrary number of qubits is inadvertently measured is discussed at the end.

The goal is to implement three successive Euler rotations $HR_z(\alpha_i)$, with the α_i unequal angles. As discussed in Sec. II A, these constitute a universal single-qubit unitary on the computational qubits. After entanglement, $R_y(\pi/2)$ is applied to all of the qubits so that future rotations can be performed only about the X axis. To effect the first two Euler rotations, one applies R_x rotations such that the first and second qubits are chosen to have a total angle of $\alpha_1 + \pi$ and $\alpha_2 + \pi$, respectively. This can be accomplished by applying two rotation pulses in analogy to the cluster-carving protocol described in Sec. III, except now only with R_x rather than R_y rotations. The first pulse rotates the first qubit by an angle θ_1 and the second pulse rotates the second qubit by θ_2 , where the θ_i can be chosen by solving the linear system

$$\alpha_1 + \pi = \theta_1 + e^{-1/r^2} \theta_2 \quad (23)$$

$$\alpha_2 + \pi = e^{-1/r^2} \theta_1 + \theta_2 \quad (24)$$

with r the width of the rotation beam. Solving for θ_1 and θ_2 one obtains

$$\theta_1 = \frac{\alpha_1 + \pi - e^{-1/r^2}(\alpha_2 + \pi)}{1 - e^{-2/r^2}}. \quad (25)$$

$$\theta_2 = \frac{\alpha_2 + \pi - e^{-1/r^2}(\alpha_1 + \pi)}{1 - e^{-2/r^2}}; \quad (26)$$

These pulses will apply undesired unitaries to the other qubits in the chain, but these can be compensated for later. The total applied rotations are then

$$\begin{aligned} 1 : & R_x(\alpha_1 + \pi)R_y(\pi/2) \\ 2 : & R_x(\alpha_2 + \pi)R_y(\pi/2) \\ 3 : & R_x(e^{-4/r^2}\theta_1 + e^{-1/r^2}\theta_2)R_y(\pi/2) \\ & \vdots \end{aligned}$$

i.e. total unitaries of $HR_z(\alpha_1)$ and $HR_z(\alpha_2)$ have now been applied to qubits 1 and 2, respectively.

Measurement of the qubit 1 teleports the logical state $X^{m_1}HR_z(\alpha_1)|\psi\rangle$ to the second qubit. Taking into account the finite probability of an inadvertent measurement of the second qubit, the logical state (defined as the state of the qubit immediately to the right of the atom just measured, not including as-yet unmeasured qubits further to the right or single-qubit unitaries that have resulted from previous rotations) is the density matrix:

$$\begin{aligned} \rho_1 &= (1-p)||X^{m_1}HR_z(\alpha_1)|\psi\rangle|| \\ &+ p||X^{m_2}HR_z(\alpha_2)X^{m_1}HR_z(\alpha_1)|\psi\rangle||, \\ &= (1-p)||X^{m_1}HR_z(\alpha_1)|\psi\rangle|| \\ &+ p||X^{m_2}Z^{m_1}R_x[(-1)^{m_1}\alpha_2]R_z(\alpha_1)|\psi\rangle||, \end{aligned} \quad (27)$$

where $||\psi\rangle|| \equiv |\psi\rangle\langle\psi|$, $p = p_f$, and the identities $R_z(\alpha)X \equiv XR_z(-\alpha)$ and $R_x(\alpha) = HR_z(\alpha)H$ have been used to derive the final expression. Note that the fidelity can never be exactly unity during the computation because there is always the probability p of inadvertently measuring the adjacent qubit.

Now the second qubit must be measured. If the second qubit had already been inadvertently projected, then the outcome of the second measurement will definitely be m_2 ; otherwise the value of m_2 will be determined presently. If $m_1 = 0$, one could carry out this measurement by leaving the first qubit in its $|0\rangle$ state and distinguishing the second qubit's measurement outcome by an increased or unchanged level of fluorescence compared to measuring the first qubit alone. In keeping with the approach outlined in the above section, however, prior to measurement one should rather apply a total rotation of π to the first qubit to rotate its state into $|1\rangle$ (which produces no fluorescence signal), and a rotation of 0 to the second qubit to preserve its state.

Neglecting for the moment the possibility of inadvertently measuring the third qubit upon measurement of the second qubit, the result would be the pure logical state $X^{m_2}Z^{m_1}R_x[(-1)^{m_1}]R_z(\alpha_1)|\psi\rangle$. If $m_1 = 0$, then the second Euler rotation will have been correctly effected, and one could simply apply an angle $\alpha_3 + \pi$ to qubit 3 to effect the last Euler rotation, completing the general single-qubit unitary. (Recall that with the wide beam one also needs to ensure that all previously intentionally measured qubits are rotated into $|1\rangle$, and that the accumulated phases on atoms beyond qubit 4 are noted for future reference). Now explicitly including the possibility of inadvertently measuring the third qubit, after measurement of qubit 2 the logical state would be

$$\begin{aligned} \rho_2 &= (1-p)||X^{m_2}R_x(\alpha_2)R_z(\alpha_1)|\psi\rangle|| \\ &+ p||X^{m_3}HR_z(\alpha_3)X^{m_2}R_x(\alpha_2)R_z(\alpha_1)|\psi\rangle|| \\ &= (1-p)||X^{m_2}R_x(\alpha_2)R_z(\alpha_1)|\psi\rangle|| \\ &+ p||X^{m_3}Z^{m_2}HR_z[(-1)^{m_2}\alpha_3]R_x(\alpha_2) \\ &\quad \times R_z(\alpha_1)|\psi\rangle||. \end{aligned} \quad (28)$$

Again, if $m_2 = 0$, then the third Euler angle will have been applied correctly, and measurement of qubit 3 will

complete the unitary. The rotation on the fourth qubit is arbitrary, so let's set it to π . The logical state is finally

$$\begin{aligned}\rho_3 &= (1-p)||X^{m_3}HR_z(\alpha_3)R_x(\alpha_2)R_z(\alpha_1)|\psi\rangle|| \\ &+ p||X^{m_4}HX^{m_3}HR_z(\alpha_3)R_x(\alpha_2)R_z(\alpha_1)|\psi\rangle|| \\ &= (1-p)||X^{m_3}HR_z(\alpha_3)R_x(\alpha_2)R_z(\alpha_1)|\psi\rangle|| \\ &+ p||X^{m_4}Z^{m_3}R_z(\alpha_3)R_x(\alpha_2)R_z(\alpha_1)|\psi\rangle||. \quad (29)\end{aligned}$$

It is important to note that the overall fidelity has not decreased from the state (27) after the first measurement. Each successive measurement effectively purifies the logical state into the same sum of two density matrices. The right-most cluster qubit is well-defined and so the very last measurement can be chosen to be unambiguous, yielding unit fidelity at the end of the full 1WQC.

The scenario above assumes that $m_i = 0$ for all i , but the probability of this occurring decreases exponentially in the number of measured qubits. Thankfully, post-selecting on these outcomes is not necessary with our protocol. Consider the case $m_1 = 1$. Measuring qubit 2 will result in the incorrect Euler angle $-\alpha_2$. This scenario is quite different from standard 1WQC, whereupon measuring $m_1 = 1$ the opposite angle would be chosen for the measurement of qubit 2 to immediately correct the feed-forward error. In the present case qubit 2 may have already been inadvertently measured, and it is too late to correct the error in this manner. Rather, measurements are performed on the next pair of qubits in an attempt to rotate the Euler angle back to the desired value. Rotations of π and $-2\alpha_2 + \pi$ are applied to qubits 3 and 4, respectively. Measuring qubits 2 and 3 then yields the logical state

$$\begin{aligned}\rho_3 &= (1-p)||X^{m_3}HX^{m_2}ZR_x(-\alpha_2)R_z(\alpha_1)|\psi\rangle|| \\ &+ p||X^{m_4}HR_z(-2\alpha_2)X^{m_3}HX^{m_2}ZR_x(-\alpha_2) \\ &\quad \times R_z(\alpha_1)|\psi\rangle|| \\ &= (1-p)||X^{m_3}Z^{m_2}XHR_x(-\alpha_2)R_z(\alpha_1)|\psi\rangle|| \\ &+ p||X^{m_4}Z^{m_3}X^{m_2}ZR_x[(-1)^{m_3}2\alpha_2]R_x(-\alpha_2) \\ &\quad \times R_z(\alpha_1)|\psi\rangle||. \quad (30)\end{aligned}$$

If $m_3 = 0$, then the logical state becomes

$$\begin{aligned}\rho_3 &= (1-p)||Z^{m_2}XHR_x(-\alpha_2)R_z(\alpha_1)|\psi\rangle|| \\ &+ p||X^{m_4+m_2}ZR_x(\alpha_2)R_z(\alpha_1)|\psi\rangle||, \quad (31)\end{aligned}$$

so that the correct second Euler would be applied upon measurement of qubit 4. It suffices to rotate qubit 5 into $\alpha_3 + \pi$ in advance, and continue as if no error had occurred. If rather $m_3 = 1$, then the applied Euler angle would be $-3\alpha_2$, and another qubit pair need to be inserted (the second with angle $4\alpha_2 + \pi$), etc. The probability of *not* obtaining a 0 outcome on the relevant odd-numbered qubit (and therefore not applying the correct Euler rotation) decreases exponentially in the number of attempts.

Following this example above, if now $m_3 = 0$ one can choose qubit 5 to have the phase $(-1)^{m_2}\alpha_3 + \pi$, so that

measurement of qubit 4 yields the logical state

$$\begin{aligned}\rho_4 &= (1-p)||X^{m_4+m_2}ZR_x(\alpha_2)R_z(\alpha_1)|\psi\rangle|| \\ &+ p||X^{m_5}HR_z[(-1)^{m_2}\alpha_3]X^{m_4+m_2}ZR_x(\alpha_2) \\ &\quad \times R_z(\alpha_1)|\psi\rangle|| \\ &= (1-p)||X^{m_4+m_2}ZR_x(\alpha_2)R_z(\alpha_1)|\psi\rangle|| \\ &+ p||X^{m_5}Z^{m_4+m_2}XHR_z[(-1)^{m_4}\alpha_3]R_x(\alpha_2) \\ &\quad \times R_z(\alpha_1)|\psi\rangle||. \quad (32)\end{aligned}$$

If $m_4 = 0$, measurement of qubit 5 yields the desired unitary; otherwise, additional pairs of qubits need to be appended to qubit 5 to rotate the Euler angle back to the desired value, as described above.

We now summarize the protocol. Each $HR_z(\alpha_i)$ operation in principle requires only one measurement on each physical qubit, all prefaced by the appropriate rotations on nearby qubits. However, if the sum of certain previous measurement outcomes is odd, one needs to correct for errors due to possible inadvertent measurements (even if they haven't occurred!) by inserting additional pairs of measured qubits. Four qubits is the minimum number for a universal single-qubit unitary, though in practice the number could be considerably higher, depending on the measurement outcomes.

While this protocol protects against inadvertent measurements on the qubit immediately to the right of the target, accidental measurement of the next-nearest neighbor could be catastrophic for the overall 1WQC. With offset measurements the probability of this occurring is small, but for a long computation the number of such events can become appreciable. A protocol protecting against inadvertent measurements of many neighbors avoids the requirement of very large laser offsets with wide beams. It could also provide a way to protect quantum information against re-absorption by distant atoms of scattered photons emanating from the fluorescing target.

To protect against more long-range inadvertent measurements requires additional 'buffer' qubits rotated by π that effect only Clifford gates. Consider the case where one intends to compensate for inadvertently measuring two neighbors. As above, all qubits initially undergo $R_y(\pi/2)$ rotations. The first two qubits undergo R_x rotations by $\alpha_1 + \pi$, $\alpha_2 + \pi$ as before. Now the possibility that the third qubit might be inadvertently projected suggests that it be rotated by π prior to any measurements. Again, the outcome of the first measurement m_1 determines if the second Euler rotation will be implemented correctly. If $m_1 = 0$, then one can proceed directly with implementing the third Euler rotation. One cannot simply rotate qubits 4 and 5 respectively by $\alpha_3 + \pi$ and π , however. The combination of π and $\alpha_3 + \pi$ on qubits 3 and 4 eliminates the Hadamard gate needed for the next Euler rotation to be around the z axis. One therefore needs qubits 4 through 6 rotated by π , $(-1)^{m_2}\alpha_3 + \pi$, and π , respectively. Again, m_4 determines the success of the last Euler rotation. If $m_4 = 0$ then qubit 7 needs to be

set to π and the fifth qubit measured to effect the single-qubit unitary. Thus, a minimum of seven qubits are required to protect against two-qubit inadvertent measurements. If either $m_1 = 1$ or $m_4 = 1$, additional qubit pairs must be added until the Euler angle is rotated back to the desired value, just as in the case considered in detail above; in the former case, for example, qubits 4 and 5 would be rotated by $-2\alpha_2 + \pi$ and π , respectively.

It should now be apparent that to protect against inadvertent measurements on m qubits to the right of the target requires on the order of $3m$ physical qubits in order to effect a universal single-qubit gate on computational qubits. Because feed-forward errors occur with probability $1/2$, the number of qubits needed to implement the correct Euler angle is of order m . A much larger number of measurements may be required in practice to correct an Euler angle error, but the probability of continued failures becomes exponentially smaller in the number of measurements.

Compensating for inadvertent measurements on multiple qubits is *not* equivalent to making actual measurements on multiple qubits. The former assumes that fluorescence from inadvertently projected qubits is not observable while the latter assumes that it is. To simultaneously measure multiple qubits one could in principle make use of the Gaussian profile of the measuring pulse: the qubit fluorescence signals are proportional to the local pulse strength. The ability to clearly identify the signal strength with an atom's position degrades rapidly with the pulse width, however, particularly considering the inherent noise associated with photon counting.

V. 2D CLUSTER

A. Scheme

The full two-dimensional protocol works much the same way as the 1D case discussed in Sec. IV. In the simplest scheme, the beams are offset horizontally as before, but now are centered vertically. The cluster must be initially carved so that pulses measuring the physical qubits on one chain have a negligible probability of projecting the states of physical qubits on an adjacent chain. The horizontal chains representing computational qubits must be sufficiently well-separated vertically. Naïvely one might assume that the chains would need to be separated by a distance at least greater than na , with n the offset defined in Eq. (21).

In practice the chains can be positioned much closer together, because of the circular spatial ‘footprint’ of the measuring pulse. Consider a beam that is centered vertically on a given chain but is offset horizontally by na . If the intensity on the first qubit is I_0 , then the first qubits on chains above and below will experience intensity $I_0 e^{-2(ma)^2/r^2}$, where ma is the separation between horizontal chains. If the desired probability of projecting the first qubit on an adjacent chain is p_m , then the chain

separation is

$$m = \frac{r}{a} \sqrt{\frac{1}{2} \ln \left[\frac{\ln(1 - p_t)}{\ln(1 - p_m)} \right]}. \quad (33)$$

Thus the inter-chain separation grows more slowly (by a factor of r/a) than the offset required to minimize inadvertent measurements on adjacent qubits on the same chain, Eq. (21). If for example one chooses $p_t = 0.99$ and a tiny value $p_m = 10^{-10}$, then $m \approx 3.5(r/a)$; with $r = 4a$ and $r = 10a$ one obtains $m \approx 14$ and $m \approx 35$, respectively. This separation is adequate for all measurements on all chains (momentarily postponing issues related to links between horizontal chains, discussed in detail below). One simply requires that all the measured qubits in the various chains have the same horizontal coordinate (column index).

Recall that prior to each measurement in the 1D protocol, one needs to perform of order $2n$ rotations. These prepare the target qubit and those to its right that might be inadvertently measured, and restore qubits previously measured and soon-to-be-remeasured to the state $|1\rangle$. In 2D, the number of measured qubits, and therefore the number of preparatory rotations, at each stage is of order πn^2 . This requires a solution to a $n^2 \times n^2$ linear system of equations whose cost scales as n^4 , a significant classical overhead. Note that the beam irradiating a given chain impinges on chains above and below. So the necessity of performing preparatory rotations implies that measurements of qubits on chains separated by distances smaller than na cannot be done in parallel, unless the solution of the relevant linear equations involves all rows of physical qubits. In practice, this apparent lack of operational parallelism is no serious impediment: in the worst-case scenario considered above with $r = 10a$, there are only 4.4 chains per full offset.

The main extension of the 1D protocol to the 2D cluster is how to implement the teleportation primitive on the links between horizontal chains, in order to simulate an entangling gate between computational qubits. As was discussed in Sec. II A, no feed-forward is required when single-qubit measurements teleport Clifford gates. Because the CZ gate belongs to the Clifford group, with suitable preparation of the relevant qubits (i.e. application of total rotation of π) the inter-chain links (hitherto referred to as simply links) should be unaffected by inadvertent measurements. That said, the outcomes of measurements on links need to be unequivocally known, since they will affect the feed-forward criteria for future measurements on chains.

Making well-defined measurements on qubits in vertically oriented links is extremely difficult, however. Consider the situation where the target qubit on a given chain is the first qubit of a vertical link. Using Eq. (9), the probability of measuring the first link qubit is close to p_t . Indeed, Eq. (33) states that m qubits along the link from the target will be measured with probability p_m or greater. If we wish that this probability be at least

smaller than the probability of inadvertently measuring a qubit to the right of the target along the chain $p_{m'} < p_f$, then the number of link qubits that will be measured is $m' = \sqrt{2n+1} \approx m/2$ using Eq. (21). Thus, the states of individual link qubits will be impossible to uniquely determine.

The solution to this apparent conundrum is to form inter-chain links that are essentially diagonal, with the ‘zig-zag’ pattern shown in Fig. 3. The first two qubits in the link must be vertically oriented. Recall that the cluster is formed by first initializing all cluster qubits to $|+\rangle$ followed by the CZ gate, and one needs to avoid forming a box graph. With this geometry, the maximum number of inadvertent measurements will be the number of qubits in the column to the right of the target. It is clear from the figure that this number will often be three: one on the chain and two nearby in the link. More importantly, the maximum number of qubits intentionally measured simultaneously will also be three. Even without the ability to spatially resolve the signal emanating from three atoms, however, the individual states of the three qubits can still be obtained with minimal operational overhead as shown explicitly below.

B. Protocol

The 2D protocol differs primarily in that simultaneous measurements of chain and link qubits must be performed. Suppose the first target is in column 1, on the upper chain just left of the first link qubit, as illustrated in Fig. 3. It is assumed to be in the state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ with α, β complex coefficients satisfying $|\alpha|^2 + |\beta|^2 = 1$. If all entangled qubits are first rotated by π , after measurement of qubit 1 the resulting logical state is

$$\begin{aligned} \rho_1 \approx & (1 - p_f - p_f^2 - p_f^3) \|X^{m_1} H |\psi\rangle\| \\ & + p_f \|X_1^{m_2} H_1 H_2 X_1^{m_1} H_1 |\tilde{\psi}\rangle\| \\ & + p_f^2 \|X_2^{m_3} H_2 X_1^{m_2} H_1 H_2 X_1^{m_1} H_1 |\tilde{\psi}\rangle\| \\ & + p_f^3 \|X_2^{m_4} H_2 X_2^{m_3} H_2 X_1^{m_2} H_1 H_2 X_1^{m_1} H_1 |\tilde{\psi}\rangle\| \\ = & (1 - p_f - p_f^2 - p_f^3) \|X^{m_1} H |\psi\rangle\| \\ & + p_f \|X_1^{m_2} Z_1^{m_1} H_2 |\tilde{\psi}\rangle\| + p_f^2 \|X_2^{m_3} X_1^{m_2} Z_1^{m_1} |\tilde{\psi}\rangle\| \\ & + p_f^3 \|X_2^{m_4} Z_2^{m_3} X_1^{m_2} Z_1^{m_1} H_2 |\tilde{\psi}\rangle\|. \end{aligned} \quad (34)$$

This expression assumes that inadvertent measurements are possible only on qubits in column 2 (labeled qubits 2, 3, and 4 in Fig. 3), in the vicinity of the upper chain. The small differences in the probabilities of inadvertently measuring qubits down the link are neglected for clarity, and are all set to p_f . Note that to define the current logical state one can ignore measurements beyond the qubit that connects to the original logical state. Also, in practice one can ignore terms with prefactors higher than p_f^2 , since these are much smaller than the probabilities of inadvertently measuring second-nearest neighbors which are ignored in the current treatment. In any case,

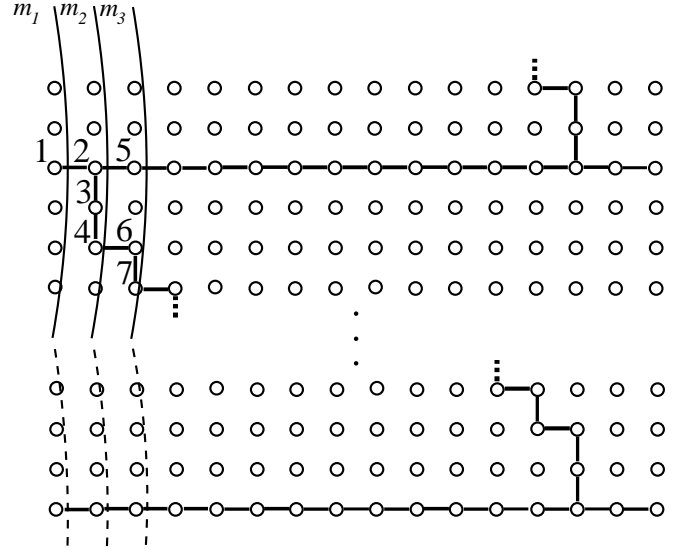


FIG. 3: The full 2D protocol. Initially, qubit 1 in the first column of the upper chain is measured by a beam that is offset horizontally but centered vertically (the right-most edge of the beam is labeled m_1). Immediately thereafter, the first qubit on the second chain is measured with the same horizontal offset, as is the first in the third chain, etc. Next, qubits 2, 3, and 4 in the second column are simultaneously measured (m_2), as is the second qubit in the lower chain, and so on. Measurements continue in this fashion until the logical state is entangled between the two chains.

the outcomes of these inadvertent measurements will be determined shortly by intentional measurements.

The most important new feature in the density matrix (34) is the appearance of the state $|\tilde{\psi}\rangle \equiv \alpha|00\rangle + \beta|11\rangle$. When a measurement is performed on the chain qubit that is also the first link qubit, the chain and link qubits become inherently entangled. The logical state must therefore be written in a two-particle basis with $|0\rangle \mapsto |00\rangle$ and $|1\rangle \mapsto |11\rangle$, with the first and second elements representing the chain and link qubits, respectively. Of course, this is how the link is able to simulate a CZ gate between two chains: upon measurement of the last link qubit, which is located on the second chain, the two qubit basis corresponds to chain 1 and 2. The subscripts on the single-particle operators in Eq. (34) therefore correspond to the qubit index in the two-particle basis.

After rotating all nearby unmeasured cluster qubits by π , the measurement of atoms in the second column (qubits 2-4 in Fig. 3) can be performed. The outcome is

$$\begin{aligned} \rho_2 \approx & (1 - 2p_f - 2p_f^2) \|X_2^{m_4} Z_2^{m_3} X_1^{m_2} Z_1^{m_1} H_2 |\tilde{\psi}\rangle\| \\ & + p_f \|X_1^{m_5} X_2^{m_4} Z_2^{m_3} Z_1^{m_2} X_1^{m_1} H_2 H_1 |\tilde{\psi}\rangle\| \\ & + p_f \|X_2^{m_6} Z_2^{m_4} X_2^{m_3} X_1^{m_2} Z_1^{m_1} |\tilde{\psi}\rangle\| \\ & + p_f^2 \|X_2^{m_7} Z_2^{m_6} X_2^{m_4} Z_2^{m_3} X_1^{m_2} Z_1^{m_1} H_2 |\tilde{\psi}\rangle\| \\ & + p_f^2 \|X_2^{m_6} X_1^{m_5} Z_2^{m_4} X_2^{m_3} Z_1^{m_2} X_1^{m_1} H_1 |\tilde{\psi}\rangle\|, \end{aligned} \quad (35)$$

ignoring terms of order p_f^3 and higher. How is it possible to uniquely determine the outcome $|m_2m_3m_4\rangle$? If no signal is obtained, the three-qubit state is definitely $|111\rangle$. If a weak signal indicates only one atom is in $|0\rangle$, then the possible outcomes are $|011\rangle$, $|101\rangle$, and $|110\rangle$. Subsequent rotation of the qubit 2 by π will either yield $|111\rangle$ or two qubits in $|0\rangle$, which can be easily verified by re-measurement of the three qubits. If the latter outcome is obtained, then re-rotating qubits 2 and 3 both by π will unequivocally determine the correct initial output. The same approach works for an original measurement indicating two $|0\rangle$ outcomes, and trivially for $|000\rangle$ or $|111\rangle$. Even if the relative signal strengths corresponding to different numbers of $|0\rangle$ outcomes cannot be distinguished, there are only seven different combinations that need to be attempted (by repeated rotations and measurements) in order to convert the output to $|111\rangle$ and thereby eliminate the fluorescence.

Measurements continue in this columnwise fashion until the link connects with the lower chain. The signal strength from link qubits on measurements of chain 1 qubits will gradually decrease, becoming undetectable soon after the midpoint between chains. Before this column is reached, measurement of chain 2 qubits will pick up signal from the link qubits, maintaining the flow of information down the link. If the fluorescence noise is too great, then one would simply move the center of the measuring beam vertically between the two chains to improve the signal.

VI. SUMMARY AND DISCUSSION

We have described in detail a protocol for deterministic quantum computation in the one-way model, where multiple physical qubits are simultaneously measured. This approach is well-suited to implementation with ultracold atoms confined in optical lattices, where the measuring beam is much wider than the separation between qubits. A central assumption of the approach is that the center of the lasers can nevertheless be positioned with high accuracy. Our strategy makes use of the Gaussian profile of lasers used in these systems for performing both rotations and measurements. For rotations, the Gaussian profile allows for a unique rotation on each cluster qubit to be performed by superimposing rotations on all physical qubits. This enables the direct generation of the desired computational cluster state, and to teleport the desired unitaries during the one-way computation. For measurements, the laser center is chosen to be far removed from the qubit of interest in order to minimize the possibility of making inadvertent measurements on nearby qubits. With additional overhead in terms of physical qubits, however, the protocol can accommodate an arbitrary number of unknown outcomes of measurements accidentally performed on nearby contiguous qubits.

Each measurement, which effects a gate on the com-

putational qubits, is performed in four steps. First, all qubits that will be irradiated by the measuring beam, as well as several qubits beyond, must first have their phases suitably prepared. This is accomplished by applying a rotation pulse centered on each physical qubit. Second, the relevant cluster qubits are measured. Third, if more than one qubit is intentionally measured, then these need to be further rotated and subsequently re-measured. Fourth, if previous measurement outcomes will incorrectly apply a future rotation due to an inadvertent measurement, then this error needs to be fixed by a finite set of future measurements.

Because teleportation errors caused by inadvertent measurements need to be fixed by extending the number of measurements of qubits in the horizontal chains, it is not known *a priori* how many chain qubits will be needed to effect the desired unitary. This is somewhat problematic because the computational cluster, i.e. the entire structure of horizontal chains and vertical links, needs to be formed in advance of any measurements. The simplest solution is to make all the inter-link distances sufficiently long that the total probability of performing the correct unitary will be high. The probability of continued failure after ℓ attempted corrections is $1/2^{\ell+1}$. If the desired probability of success is $1 - \epsilon$ then the number of correction attempts must be $\ell > \log_2(1/\epsilon) - 1$. In order to protect against inadvertent measurements of m neighbors, an arbitrary unitary will require a chain of $3(lm + 1)$ qubits for even m or $3[l(m + 1) + 1]$ for odd m . Once the correct unitary is accomplished, measuring the remaining qubits will teleport Clifford gates with trivial consequences.

An attractive feature of this scheme is that in principle it could partially mitigate the second most significant impediment in the implementation of 1WQC with ultracold atoms in optical lattices: stray fluorescence from measurements accidentally projecting distant qubits. Because photons are emitted in all directions, the probability of inadvertently measuring qubits a distance d from the target decreases like d^{-2} . Simply buffering the chains with 32 extra qubits (each rotated by π to teleport Clifford gates) reduces the deleterious effects of stray light by a factor over 1000.

While the 1WQC scheme outlined in this manuscript can account for the possibility of having inadvertently measured qubits, it is currently not designed to protect against the small but finite possibility that the *target* is not actually projected by the measurement. A central assumption throughout has been that the laser coupling, measurement time, and the decay rate are all sufficiently large that an intentional measurement is performed with high certainty. The issue might seem irrelevant because any given target is consistently re-measured as the protocol moves down a chain or inter-chain link, so it will quickly become projected if it was not initially, and any error in assumptions will be made manifest. The problem is that rotating a qubit that is believed to have been already projected, but is in fact entangled with

yet-unmeasured cluster qubits, will possibly teleport an unknown Pauli byproduct gate on subsequent measurements. This issue is however beyond the scope of the current calculations, and will be the focus of future work.

Acknowledgments

The authors are grateful to Mark Tame and Terry Rudolph for stimulating discussions in the early stages

of the work, and to Michael Garrett for insightful comments. This research was supported by the Natural Sciences and Engineering Research Council of Canada and the Canada Foundation for Innovation.

-
- [1] H.-J. Briegel and R. Raussendorf, Phys. Rev. Lett. **86**, 910 (2001).
 - [2] P. Walther, K. J. Resch, T. Rudolph, E. Schenck, H. Weinfurter, V. Vedral, M. Aspelmeyer, and A. Zeilinger, Nature **434**, 169 (2005).
 - [3] M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature **415**, 39 (2002).
 - [4] M. Köhl, H. Moritz, T. Stöferle, C. Schori, and T. Esslinger, J. Low Temp. Phys. **138**, 635 (2005).
 - [5] G. K. Brennen, C. M. Caves, P. S. Jessen, and I. H. Deutsch, Phys. Rev. Lett. **82**, 1060 (1999).
 - [6] D. Jaksch, J. I. Cirac, P. Zoller, S. L. Rolston, R. Cote, and M. D. Lukin, Phys. Rev. Lett. **85**, 2208 (2000).
 - [7] L.-M. Duan, E. Demler, and M. D. Lukin, Phys. Rev. Lett. **91**, 090402 (2003).
 - [8] S. Stock, Z. Hadzibabic, B. Battelier, M. Cheneau, and J. Dalibard, Phys. Rev. Lett. **95**, 190403 (2005).
 - [9] I. B. Spielman, W. D. Phillips, and J. V. Porto, cond-mat/0606216v2 (2007).
 - [10] D. Jaksch, H.-J. Briegel, J. I. Cirac, C. W. Gardiner, and P. Zoller, Phys. Rev. Lett. **82**, 1975 (1999).
 - [11] O. Mandel, M. Greiner, A. Widera, T. Rom, T. W. Hänsch, and I. Bloch, Phys. Rev. Lett. **91**, 010407 (2003).
 - [12] L.-M. Duan, E. Demler, and M. D. Lukin, Phys. Rev. Lett. **91**, 090402 (2003).
 - [13] A. B. Kuklov and B. V. Svistunov, Phys. Rev. Lett. **90**, 100401 (2003).
 - [14] S. Peil, J. V. Porto, B. Laburthe Tolra, J. M. Obrecht, B. E. King, M. Subbotin, S. L. Rolston, and W. D. Phillips, Phys. Rev. A **67**, 051603(R) (2003).
 - [15] R. Scheunemann, F. S. Cataliotti, T. W. Hänsch, and M. Weitz, Phys. Rev. A **62**, 051801(R) (2000).
 - [16] D. S. Weiss, J. Vala, A. V. Thapliyal, S. Myrgren, U. Vazirani, and K. B. Whaley, Phys. Rev. A **70**, 040302(R) (2004).
 - [17] T. Calarco, U. Dorner, P. S. Julienne, C. J. Williams, and P. Zoller, Phys. Rev. A **70**, 012306 (2004).
 - [18] A. Kay and J. K. Pachos, New J. Phys. **6**, 126 (2004).
 - [19] A. Kay, J. K. Pachos, and C. S. Adams, Phys. Rev. A **73**, 022310 (2006).
 - [20] C. Zhang, S. L. Rolston, and S. DasSarma, Phys. Rev. A **74**, 042316 (2006).
 - [21] J. Joo, Y. L. Lim, A. Beige, and P. L. Knight, Phys. Rev. A **74**, 042344 (2006).
 - [22] J. Cho, Phys. Rev. Lett. **99**, 020502 (2007).
 - [23] D. Walls and G. Milburn, *Quantum Optics* (Springer, Berlin, 1994).
 - [24] R. Blatt and P. Zoller, European Journal of Physics **9**, 250 (1988).
 - [25] M. B. Plenio and P. L. Knight, Reviews of Modern Physics **70**, 101 (1998).
 - [26] M. J. Gagen and G. J. Milburn, Phys. Rev. A **47**, 1467 (1993).
 - [27] D. Leibfried, R. Blatt, C. Monroe, and D. Wineland, Reviews of Modern Physics **75**, 281 (2003).
 - [28] A. Baker, *Transcendental Number Theory* (Cambridge University Press, 1990).
 - [29] D. E. Browne and H. J. Briegel, quant-ph/0603226 (2006).